

- 1) Complete the square of the following equation and factor. That is, complete the square and then write the equation in the form $(x + \frac{b}{2})^2 = c$. Do not solve the equation.

$$x^2 + 8x - 5 = 0 \rightarrow x^2 + 8x = 5 \rightarrow \text{need this form to complete the square}$$

$$\text{add } (\frac{b}{2})^2 = (\frac{8}{2})^2 = 16 \text{ to both sides}$$

$$x^2 + 8x + 16 = 16 + 5 \rightarrow \text{factor}$$

$$(x + 4)^2 = 21$$

- 2) Solve the following equation:

$$|4x - 2| = 6$$

$$4x - 2 = 6 \quad \text{or} \quad 4x - 2 = -6$$

$$4x = 8 \quad \text{or} \quad 4x = -4$$

$$\boxed{x=2} \quad \text{or} \quad \boxed{x=-1}$$

1.7

- 1) Determine the values for which the following inequality is satisfied. Write your answer in interval notation

$$\frac{3x - 3}{x + 2} \geq 0$$

> 0
 < 0

Numerator = 0 when $x = 1$

Denominator = 0 when $x = -2$

$x = -3: \frac{-12}{-1} = 12 > 0$

$x = 2: \frac{3}{4} > 0$

$x = 0: -\frac{3}{2} < 0$

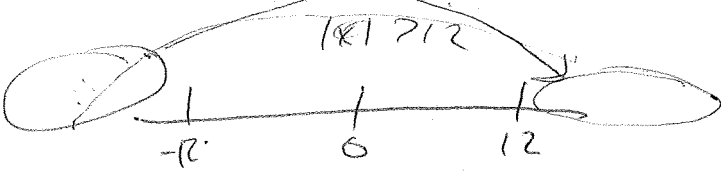
$(-\infty, -2), (-2, 1), (1, \infty)$
 ↓ ↓ ↓
 $-3 \quad 0 \quad 2$

↳ test values

$(-\infty, -2) \cup [1, \infty)$

- 2) Determine the values for which the following inequality is satisfied. Write your answer in interval notation

$$|x + 6| > 12$$



$$x + 6 > 12 \quad \text{or} \quad x + 6 < -12$$

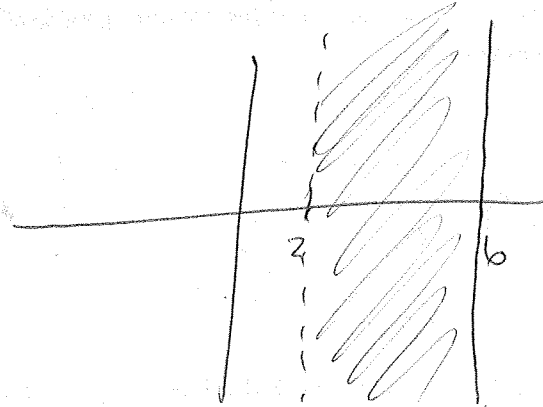
$x > 6 \quad \text{or} \quad x < -18$

$x \in \mathbb{R}$
 $(-\infty, -18) \cup (6, \infty)$

$x = 6$
 $||21 = 12 \neq 12$

3) Graph the region describe below.

$$\{(x, y) | 2 < x \leq 6\}$$



4) Find the X and Y intercepts of the following equation:

$$x^3 + 2y = 8$$

X-int -

$$\text{Set } y = 0$$

$$x^3 = 8$$

$$\Rightarrow \boxed{x = 2}$$

Y-int
Set $x = 0$

$$2y = 8$$

$$\boxed{y = 4}$$

- 1) Find the domain and range of the following function. Express your answer in Interval Notation

$$f(x) = \sqrt{4 + x}$$

Domain: $\begin{array}{l} \text{need } 4+x \geq 0 \\ \Rightarrow x \geq -4 \end{array}$

$$[-4, \infty)$$

Range: $\sqrt{\text{anything}} \geq 0$

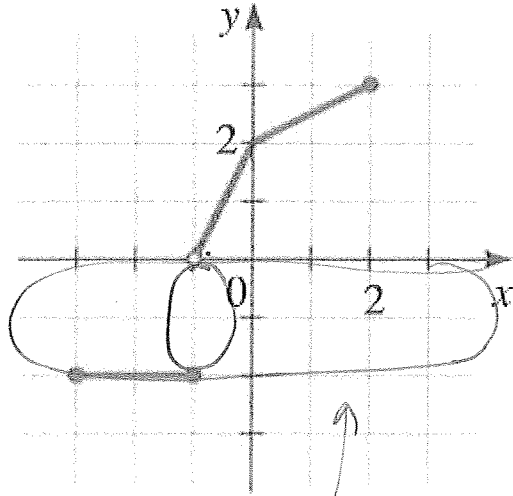
$$[0, \infty)$$

- 2) Write down and simplify $f(a+h)$

plug in $a+h$ into x in the above function

$$F(a+h) = \sqrt{4 + a+h}$$

Consider the graph shown to the left to answer the following questions



No y values
between -2 and 0

1) Write the range using interval notation

"Where are the y-values?" - $\{-2\} \cup (0, 3]$

2) Write the domain using interval notation

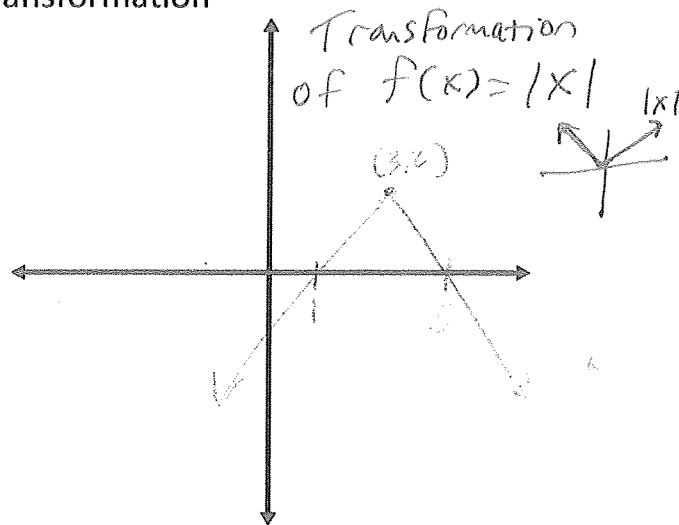
"Where are the x-values?" - $[-3, 2]$

3) $f(-1) = -2$

4) Using the vertical line test from section 2.2, is the graph depicting a function? Explain...

No matter where you draw a vertical line it only touches the graph once. This shows how each x-value corresponds to a single y-value which is the definition of a function.

- 1) Write the equation for the following transformation



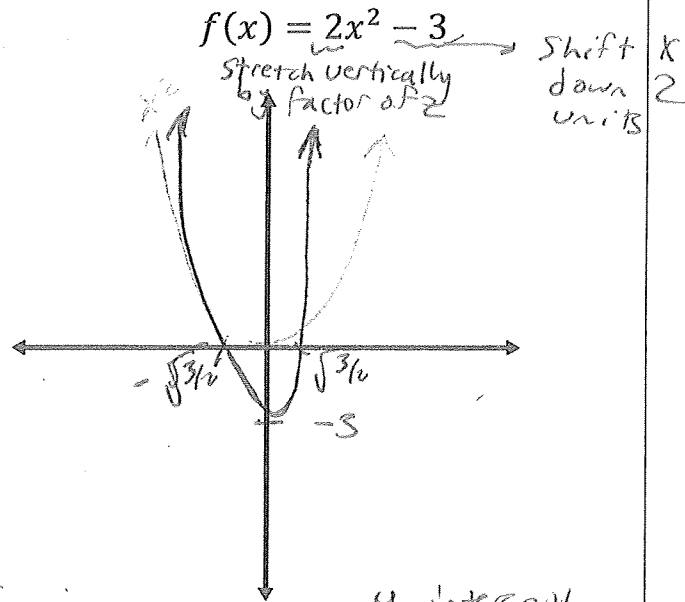
$$f(x) = -|x-3| + 2$$

Flips $|x|$ upside down

Shifts $|x|$ right 3 units

Moves $|x|$ up 2 units

- 2) Graph the following transformation of x^2



x Intercepts

$$\text{Set } y=0: 0 = 2x^2 - 3$$

$$2x^2 = 3$$

$$x^2 = \frac{3}{2}$$

$$x = \pm\sqrt{\frac{3}{2}}$$

y intercept

$$\text{Set } x=0: y = -3$$

Let $f(x) = \frac{1}{\sqrt{x}}$ and $g(x) = x - 2$

1) Find $f(g(x))$

$$f(g(x)) = \frac{1}{\sqrt{x-2}}$$

↳ plug $g(x)$ into the x in $f(x)$

2) Find $g(f(x))$

$$g(f(x)) = \frac{1}{\sqrt{x}} - 2$$

↳ plug $f(x)$ into the x in $g(x)$

3) Calculate $g(f(4))$ and simplify your answer

$$g(f(4)) = \frac{1}{\sqrt{4}} - 2 = \frac{1}{2} - 2 = \frac{1}{2} - \frac{4}{2} = -\frac{3}{2}$$



- 1) Is the function $f(x) = |x|$ a one-to-one function? If so, explain how you know. If not, provide a counterexample illustrating why it is not one-to-one.

One-to-one functions have each y -value corresponding to a single x -value

$f(x) = |x|$ is not one-to-one since $f(-1) = f(1) = 1 \rightarrow 2$ x -values correspond to same y

In general, $f(-x) = f(x) = |x|$. This can be seen by noting how the graph fails the horizontal line test.

- 2) Use the inverse function property to show that the following functions are each other's

inverse: $f(x) = \frac{3x-5}{5}$ and $g(x) = \frac{5}{3}x + \frac{5}{3}$

check $g(f(x))$

$$g(f(x)) = \frac{5}{3} \left(\frac{3x-5}{5} \right) + \frac{5}{3} = \frac{1}{3}(3x-5) + \frac{5}{3}$$

$$= x - \frac{5}{3} + \frac{5}{3} = x$$

also $f(g(x)) = x$

Sketch the graph of the polynomial function. Make sure your graph shows all intercepts and exhibits the proper end behavior.

Factor to see where $f(x) = 0$

$$f(x) = x^4 + 4x^3$$

① $f(x) = x^3(x+4) = 0$
if $x=0, x=-4$

② Test points between 0 and -4 to see if the graph is positive or negative

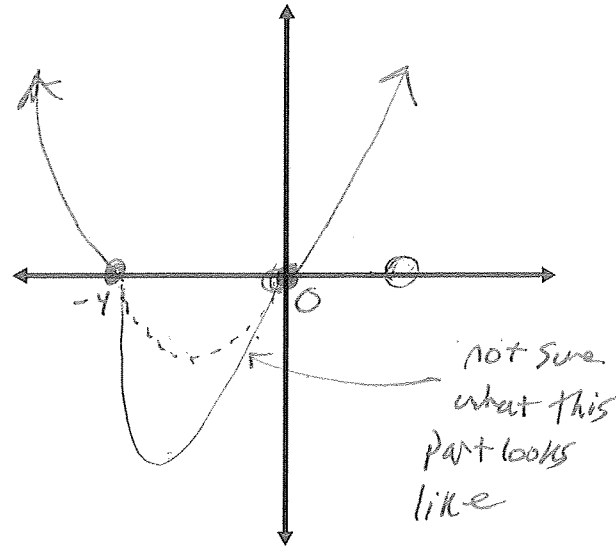
I choose $x = -1$: $f(-1) = (-1)^4 + 4(-1)^3 = -3$

\Rightarrow graph negative between 0 and -4

③ End behavior:

$f(x)$ is degree 4 (even) so both ends point in the same direction

Since the x^4 term is positive, the ends point up



For each of the following two functions, find the X-intercept, Y-intercept, horizontal asymptotes and vertical asymptotes. Show your work.

a) $\frac{5x^0}{x-2}$ For X-intercept
 Set $y=0$: $0 = \frac{5}{x-2}$
 true if $0=5$ → doesn't make sense
 So there is no X-intercept

For Y-intercept: Set $x=0$: $y = \frac{5}{-2}$ → y intercept

Horizontal Asymptote: $n=0, m=1. n < m \Rightarrow$ case 1
 case 1 \Rightarrow Hor. Asy: at $y=0$

Vertical Asymptote: Set denominator $= 0$
 $x-2=0$ if $x=2$

X-Int: Does not exist (NONE)

Y-Int: $(0, \frac{5}{-2})$

Horizontal Asy.: $y=0$

Vertical Asym: $x=2$

b) $\frac{16x^2-1}{(2x-8)(x+1)}$ X int: $16x^2-1=0$
 $x^2 = 1/16$
 $x = \pm \sqrt{1/16} = \pm \frac{1}{4}$
 Y int: $x=0: \frac{-1}{(-8)(1)} = \frac{1}{8}$

Hor asym: $n=2, m=2$
 \Rightarrow case 2 \Rightarrow Hor asy at
 $y = \frac{16}{2} = 8$

Vert asym: $(2x-8)(x+1)=0$
 if $x=4$ or $x=-1$

X-Int: $(\pm \frac{1}{4}, 0)$

Y-Int: $(0, 1/8)$

Horizontal Asy.: $y=8$

Vertical Asym: $x=4, x=-1$

- 1) From 4.1&4.2: Suppose $y = a^x$ for some base a and that the function passes through the point (3,27). Find the value of a .

(x,y) What raised to the 3rd power is 27?

$$27 = a^3 \rightarrow A: 3 \Rightarrow \boxed{a=3}$$

- 2) From 4.3: Express the following equation in exponential form: $\log_3 81 = 4$

$$\log_a x = y \Leftrightarrow a^y = x$$

$$\text{So } \log_3 81 = 4 \Leftrightarrow 3^4 = 81$$

- 3) Evaluate $\log_{100} 1 =$

$$100^x = 1 \rightarrow 100 \text{ raised to what power is } 1? \Rightarrow 0$$

$$\Rightarrow \boxed{x=0}$$

- 4) Use the definition of a logarithm to find x : $\log_4 2 = x$

$$4^x = 2 \rightarrow 4 \text{ raised to what power is } 2?$$

$$\sqrt{4} = 2 \Rightarrow 4^{1/2} = 2 \Rightarrow \boxed{x=1/2}$$

- 1) Use the Laws of Logarithms to combine the expression. Simplify your answer.

$$\begin{aligned} \ln 4 + \frac{1}{2} \ln 16 - \ln 2 &= \ln 4 + \ln(\sqrt{16}) - \ln 2 \\ &= \ln(4 \cdot \sqrt{16}) - \ln(2) = \ln\left(\frac{4 \cdot \sqrt{16}}{2}\right) = \ln(8) \\ &\quad \text{or } \log_e(8) \end{aligned}$$

- 2) Use the Laws of Logarithms to expand the expression and eliminate any exponents.

$$\begin{aligned} \log \frac{x^3 y^4}{z^6} &= \log(x^3 y^4) - \log(z^6) \\ &= \log(x^3) + \log(y^4) - \log(z^6) \\ &= \boxed{3 \log(x) + 4 \log(y) - 6 \log(z)} \end{aligned}$$

- 3) Solve the following equation for x. You should get a whole number as an answer.

$e^{8-4x} = 1$ → take log of both sides. Any log works but ln is best

$$\ln(e^{8-4x}) = \ln(1)$$

$$8-4x = 0$$

$$8 = 4x$$

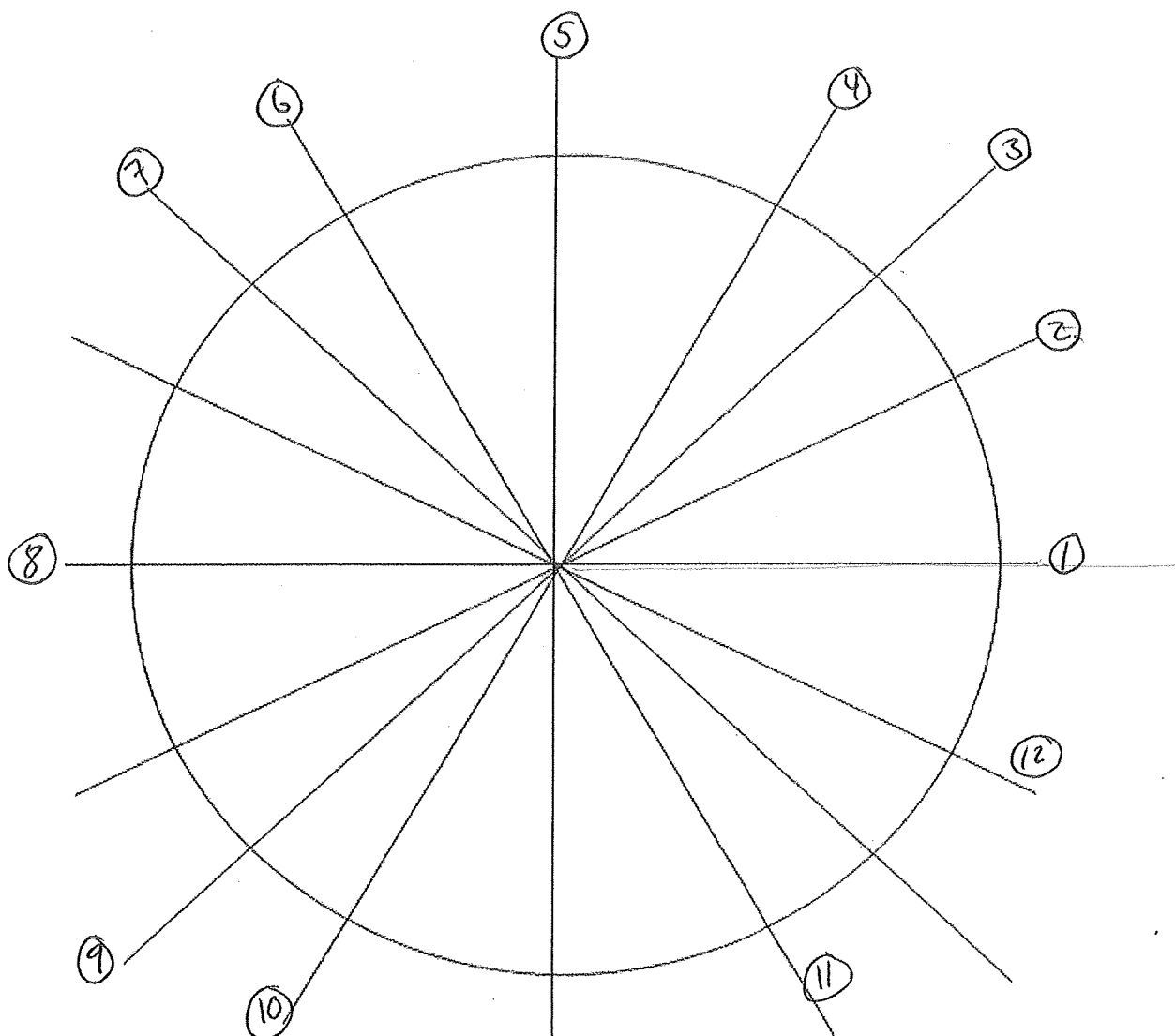
$$\boxed{2 = x}$$

check: $e^{8-4 \cdot 2} = e^{8-8} = e^0 = 1$ ✓

Unit Circle Quiz

Write your name on back

Consider the following unit circle and fill in the 12 missing pieces at the bottom of the page. Some questions ask for t values, some ask for (x,y) values and some require both.



$$\textcircled{1} \quad t = 0, 2\pi$$

$$(x,y) = (1,0)$$

$$\textcircled{4} \quad t = \pi/3 \text{ or } -5\pi/3$$

$$(x,y) = (\frac{1}{2}, \frac{\sqrt{3}}{2})$$

$$\textcircled{7} \quad t = \frac{3\pi}{4} \text{ or } -5\pi/4$$

$$(x,y) = (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$$

$$\textcircled{10} \quad t = \frac{4\pi}{3} \text{ or } -2\pi/3$$

$$(x,y) = (-\frac{1}{2}, -\frac{\sqrt{3}}{2})$$

$$\textcircled{2} \quad t = \pi/6, \frac{11\pi}{6}$$

$$(x,y) = (\frac{\sqrt{3}}{2}, \frac{1}{2})$$

$$\textcircled{5} \quad t = \pi/2 \text{ or } -3\pi/2$$

$$(x,y) = (0,1)$$

$$\textcircled{8} \quad t = \pi \text{ or } -\pi$$

$$(x,y) = (-1,0)$$

$$\textcircled{11} \quad t = \frac{5\pi}{3} \text{ or } -\pi/3$$

$$(x,y) = (\frac{1}{2}, -\frac{\sqrt{3}}{2})$$

$$\textcircled{3} \quad t = \pi/4, -7\pi/4$$

$$(x,y) = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$$

$$\textcircled{6} \quad t = \frac{2}{3}\pi \text{ or } -4/3\pi$$

$$(x,y) = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$$

$$\textcircled{9} \quad t = \frac{5\pi}{4} \text{ or } -3\pi/4$$

$$(x,y) = (-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$$

$$\textcircled{12} \quad t = \frac{11\pi}{6} \text{ or } -\pi/6$$

$$(x,y) = (\frac{\sqrt{3}}{2}, -\frac{1}{2})$$

Section 5.2 Quiz

Write your name on the back

- 1) Find the exact values of $\cos\left(\frac{5\pi}{6}\right)$ and $\tan\left(\frac{5\pi}{6}\right)$.

$\frac{5\pi}{6}$ has terminal point $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
 (x, y)

$$\cos\left(\frac{5\pi}{6}\right) = x = -\frac{\sqrt{3}}{2} \quad \tan\left(\frac{5\pi}{6}\right) = \frac{y}{x} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{-\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

- 2) Determine the sign of $(\sin t \cdot \cos t)$ given that t is in Quadrant II

$\sin t = y$, $\cos t = x$. In quad II, $x < 0$, $y > 0$

So $\sin t \cdot \cos t$ in quad II is $+ \cdot - = -$ (negative)

- 3) Write $\tan t$ in terms of $\sin t$ given that t is in Quadrant IV. Hint: $\cos t = \pm\sqrt{1 - \sin^2 t}$

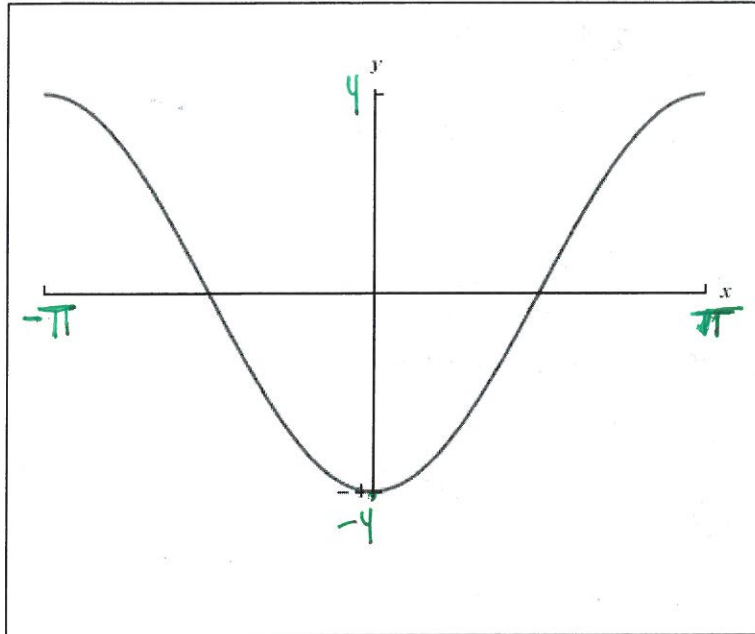
$\tan t = \frac{\sin t}{\cos t} = \pm \frac{\sin t}{\sqrt{1 - \sin^2 t}}$. In quad IV, $\cos > 0$ so use $+\sqrt{1 - \sin^2 t}$

$$\Rightarrow \tan t = \frac{\sin t}{+\sqrt{1 - \sin^2 t}}$$

Section 5.3 Quiz

Name

- 2) Determine the period, amplitude and phase shift of the following graph. You do not need to write the equation of the graph.



Period- 2π

(distance between start and end of one period)

Amplitude- 4

(How high the graph is above/below the midpoint)

Phase Shift- $-\pi$

(usually \cos starts at the y-axis. Here, it starts π to the left of the y-axis)

Section 5.3 Quiz

Name

- 1) Determine the Equation of the following graph. Include the period, amplitude and phase shift.

Period- $2 \Rightarrow \frac{2\pi}{k} = 2 \Rightarrow k = \pi$

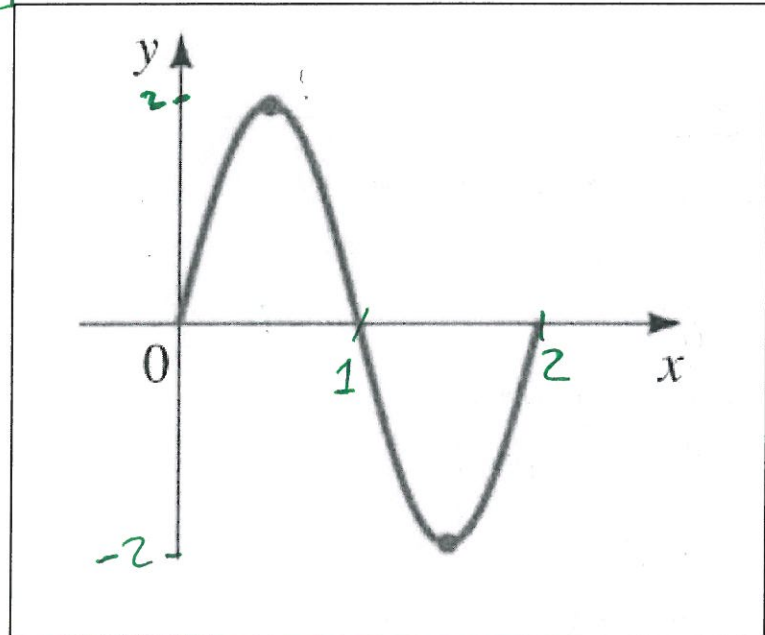
(Distance between the start and end of one period)

Amplitude- $2 \Rightarrow a = 2$

(How high the graph is above/below the midpoint)

Phase Shift- 0 / none $\Rightarrow b = 0$

(Sin usually starts at the origin)



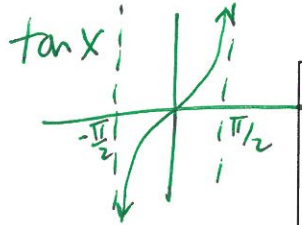
$$a \sin(k(x-b)) = 2 \sin(\pi(x-0)) = 2 \sin(\pi x)$$

Section 5.4 Quiz

Write your name on the back

1) Graph the following equation. Include the period and phase shift

$$2 \tan\left(x + \frac{\pi}{4}\right)$$



Period

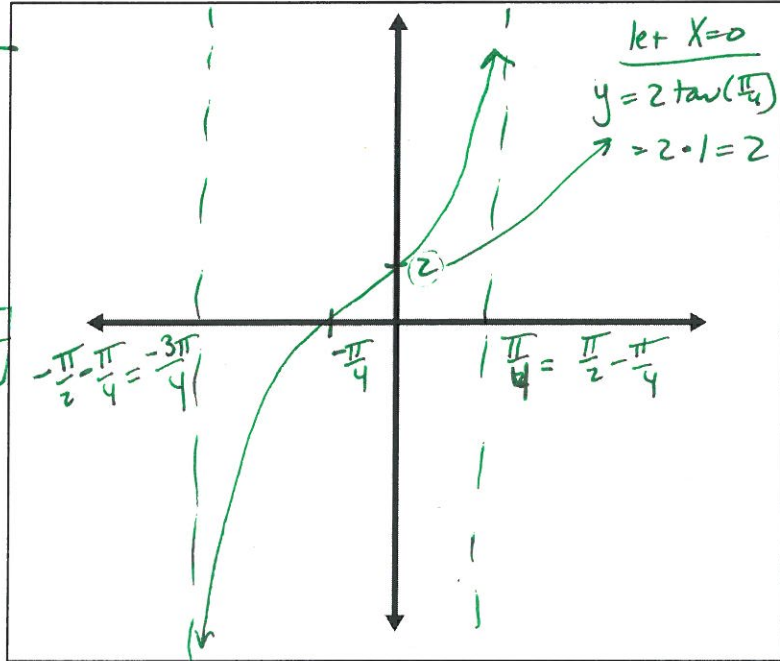
For $\tan kx$, Period = $\frac{\pi}{k}$

here, $k=1$ so Period = $\frac{\pi}{k} = \frac{\pi}{1} = \pi$

Phase Shift

$$-\frac{\pi}{4}$$

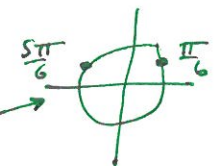
ie, $\frac{\pi}{4}$ units to the left
 \Rightarrow everything moves w/it




Section 5.5 Quiz

Write your name on the back

1) $\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$

$\sin t = y = \frac{1}{2}$ where on the unit circle? 
 need $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ so choose $\frac{\pi}{6} = t$

2) $\cos^{-1}(1) = 0$

$\cos t = x = 1$ where on the unit circle? 
 $t = 0, 2\pi, 4\pi$ etc
 need $t \in [0, \pi]$ so choose $t = 0$

3) $\tan^{-1}(\frac{\sqrt{3}}{2}) =$

$\tan^{-1}(-\frac{1}{\sqrt{3}}) = -\frac{\pi}{6}$

$\tan t = \frac{y}{x} = -\frac{1}{\sqrt{3}}$ where on the unit circle?
 need $y = x \cdot -\frac{1}{\sqrt{3}} \Rightarrow y = \frac{1}{2}, x = \frac{\sqrt{3}}{2}$
 occurs at $t = -\frac{\pi}{6}$

4) $\sin^{-1}(\sin(\frac{\pi}{2})) = \frac{\pi}{2}$ provided $\frac{\pi}{2} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\frac{\pi}{2}$ is in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

so $\sin^{-1}(\sin(\frac{\pi}{2})) = \frac{\pi}{2}$

5) $\cos(\cos^{-1}(4)) = 4$ provided $4 \in [-1, 1]$.

$4 \notin [-1, 1]$ so

$\cos(\cos^{-1}(4))$ Does not exist

6) $\tan(\tan^{-1}(\pi e^2)) = \pi e^2$ provided

$\pi e^2 \in \mathbb{R}$

i.e., πe^2 is a real number.

$\pi e^2 \in \mathbb{R}$ so $\tan(\tan^{-1}(\pi e^2)) = \pi e^2$

- 1) Convert $\frac{\pi}{9}$ rad to a measurement in degrees.

$$\frac{\pi}{9} \text{ rad} \cdot \frac{180^\circ}{\pi \text{ rad}} = \frac{180^\circ}{9} = 20^\circ$$

- 2) Convert 3° to a measurement in radians.

$$3^\circ \cdot \frac{\pi \text{ rad}}{180^\circ} = \frac{3\pi}{180} \text{ rad} = \frac{\pi}{60}$$

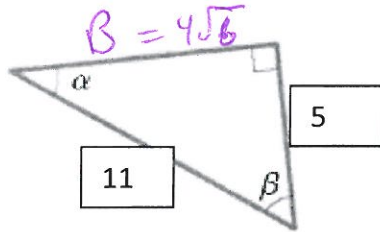
- 3) Find the length of an arc that subtends a central angle of 3° in a circle of radius 10.

$S = r\theta \rightarrow$ to use this need θ in radians

$$3^\circ = \frac{\pi}{60} \text{ rad from above}$$

So $S = r\theta = 10 \cdot \frac{\pi}{60} = \frac{\pi}{6}$

- 1) Find $\sin \beta$ and $\sin \alpha$ as they pertain to the triangle shown



Find 3rd side:

$$S^2 + B^2 = 11^2$$

$$B^2 = 121 - 25$$

$$B = \sqrt{96} = 4\sqrt{6}$$

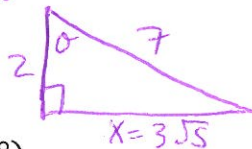
$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\text{So } \sin \alpha = \frac{5}{11}$$

$$\sin \beta = \frac{\sqrt{96}}{11} = \frac{4\sqrt{6}}{11}$$

- 2) Sketch a right triangle with acute angle θ given that $\sec \theta = \frac{7}{2}$

$$\sec \theta = \frac{\text{Hyp}}{\text{adj}} \quad \text{So Hyp} = 7, \text{ adj} = 2$$



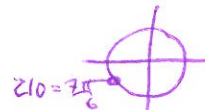
$$2^2 + X^2 = 7^2 \Rightarrow X = \sqrt{45} = 3\sqrt{5}$$

- 3) Find $\cos(210^\circ)$

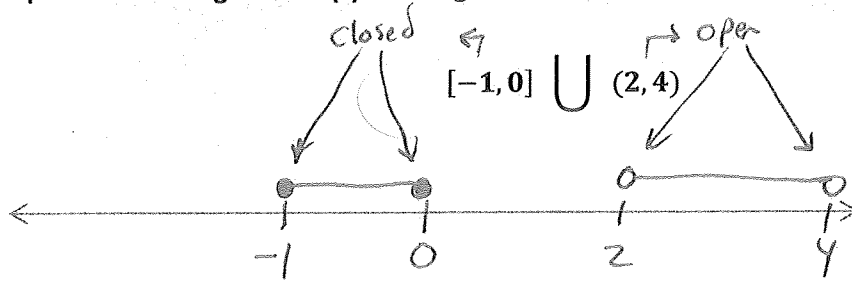
change 210° to radians:

$$210^\circ \cdot \frac{\pi}{180^\circ} = \frac{21\pi}{18} = \frac{7\pi}{6}$$

$$\text{So } \cos(210^\circ) = \cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$



- 1) Graph the following interval(s) on the given number line:



- 2) Represent the following interval in set-builder notation: $(0, 12]$

$$\{ x \mid 0 < x \leq 12 \}$$

Key
5/5

5/5